Hadronic matrix elements for B-mixing in the Standard Model and beyond

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Abstract. We use lattice QCD to calculate the B-mixing hadronic matrix elements for a basis of effective four-quark operators that spans the space of all possible contributions in, and beyond, the Standard Model. We present results for the SU(3)-breaking ratio ξ and discuss our ongoing calculation of the mixing matrix elements, including the first calculation of the beyond the Standard Model matrix elements from unquenched lattice QCD.

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MOTIVATION

Through a combination of the GIM mechanism, Cabibbo suppression, and loop suppression, the Standard Model (SM) contribution to B-mixing is small and new physics (NP) effects may be discernible [1]. In fact, there are experimental hints this may be the case. In unitarity triangle analyses [2, 3] a persistent $2-3\sigma$ inconsistency is suggestive of NP and points to B-mixing as a possible source. D0's same-sign dimuon charge asymmetry [4] and global analyses by UTfit [5] and CKMfitter [6] reveal $2-4\sigma$ discrepancies in SM B-mixing. Experimental measurements of the B-mixing oscillation frequency [7] have sub-percent precision but cannot be fully leveraged in the search for NP as theory errors, dominated by hadronic uncertainty, are an order of magnitude larger [8].

CALCULATION

To lowest order in the SM, B-mixing is described by box diagrams [*cf.* Fig. 1 (*left*)]. Under the operator product expansion (OPE) flavor-changing short-distance interactions, of O(100 GeV) in the SM and higher in NP scenarios, and long-distance hadronic physics, of O(500 MeV), factorize. At the energies relevant to B-mixing, of O($M_B \sim 5$ GeV), flavor-changing physics is described by the local, effective, four-quark interaction of Fig. 1 (*right*). A commonly used basis of mixing operators is

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{b}^\alpha \gamma_\mu L q^\alpha)\,(\bar{b}^\beta \gamma_\mu L q^\beta) & \qquad \mathcal{O}_4 &= (\bar{b}^\alpha L q^\alpha)\,(\bar{b}^\beta R q^\beta) \\
\mathcal{O}_2 &= (\bar{b}^\alpha L q^\alpha)\,(\bar{b}^\beta L q^\beta) & \qquad \mathcal{O}_5 &= (\bar{b}^\alpha L q^\beta)\,(\bar{b}^\beta R q^\alpha) \\
\mathcal{O}_3 &= (\bar{b}^\alpha L q^\beta)\,(\bar{b}^\beta L q^\alpha) & &
\end{aligned} \tag{1}$$

where L/R are left/right projection operators and α, β are color indices. A calculation of the matrix elements of these operators is sufficient to parameterize the hadronic

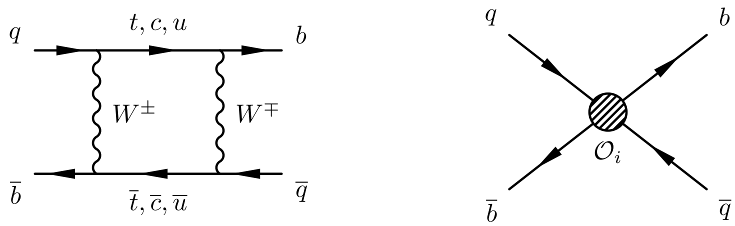

FIGURE 1. (*Left*) A SM contribution to B-mixing and (*right*) an effective four-quark interaction.

contributions to B-mixing in and beyond the SM.

Under the OPE, the expression for the oscillation frequency factorizes:

$$\Delta M_q = \sum_i C_i(\mu)\ \langle B_q^0|\mathscr{O}_i(\mu)|\overline{B}_q^0\rangle, \tag{2}$$

where the short-distance C_i are model dependent and the long-distance hadronic mixing matrix elements $\langle B_q^0|\mathscr{O}_i|\overline{B}_q^0\rangle$ must be calculated nonperturbatively using lattice QCD. The phenomenologically-relevant bag parameters $B_{B_q}^{(i)}$ and SU(3)-breaking ratio ξ are related to the matrix elements by

$$\langle B_q^0|\mathscr{O}_i|\overline{B}_q^0\rangle = \mathfrak{c}_i f_{B_q}^2 B_{B_q}^{(i)} \qquad \text{and} \qquad \xi = f_{B_s}\sqrt{B_{B_s}^{(1)}}\,/\,f_{B_d}\sqrt{B_{B_d}^{(1)}}, \tag{3}$$

where f_{B_q} is the B-meson decay constant and $\mathfrak{c}_i$ are numerical factors.[1]

The Lattice Calculation. Working in the B-meson rest-frame, we generate correlation function data via Monte Carlo evaluation of the path integral representations of the vacuum expectation values

$$C^{\text{2pt}}(t) = \langle B_q^0(t) B_q^0(0)^\dagger\rangle \qquad \text{and} \qquad C_i^{\text{3pt}}(t_1,t_2) = \langle B_q^0(t_2)\mathscr{O}_i(0)B_q^0(t_1)\rangle, \tag{4}$$

with gauge field integration performed with the MILC ensembles. We simulate with staggered light and Fermilab heavy quarks (for details of gluon and quark discretizations see [9] and references therein). Using Bayesian fitting techniques [10], these data are fit to the ansätze [11],

$$\begin{aligned} C^{\text{2pt}}(t) &= \sum_{n=0}^{N} Z_n^2(-1)^{n(t+1)}\left(e^{-M_n t} + e^{-M_n(T-t)}\right) \\ C_i^{\text{3pt}}(t_1,t_2) &= \sum_{n,m=0}^{N} \langle B_q^0|\mathscr{O}_i|\overline{B}_q^0\rangle \frac{Z_n Z_m}{2\sqrt{M_n M_m}}(-1)^{n(t_1+1)+m(t_2+1)}\ e^{-M_n t_1 - M_m t_2}\ , \end{aligned} \tag{5}$$

where Z_n is the amplitude and M_n the mass of the B-meson n^{th} excited state and T the temporal extent of the lattice. From these fits we extract the matrix elements over

[1] An alternate definition of the bag parameter includes a factor of $M_{B_q}^2/(m_b+m_q)^2$ with $\mathfrak{c}_i$ for $i \neq 1$.

a range of valence-quark masses, light sea-quark masses, and lattice spacings. Lattice matrix elements are matched to the continuum at one loop in tadpole-improved lattice perturbation theory [12]. The continuum matrix elements corresponding to physical light (for B_d^0) or strange (for B_s^0) valence quark, and at physical light sea-quark mass, are then obtained by extrapolation/interpolation with the aid of rooted, staggered, chiral perturbation theory [13, 14].

Status and Outlook. We recently completed a calculation of $\xi = 1.268(63)$ using data at lattice spacings down to ≈ 0.09 fm and valence quarks as light as $0.1\,m_s$ [15].

An ongoing calculation includes an update of ξ and the calculation of matrix elements and bag parameters for all five operators in Eq. (1). In addition, it includes several improvements: a three-fold increase in statistics; data at lattice spacings as small as ≈ 0.045 fm and valence quarks as light as $0.05\,m_s$, to reduce the effect of the continuum-chiral extrapolation; and a more thorough treatment of chiral perturbation theory [14].

Preliminary results for the matrix elements with the new, extended data set can be found in Table 4 of Ref. [16]. These results are based on data at lattice spacings down to ≈ 0.06 fm and valence quarks as light as $0.1\,m_s$. Initial studies show that our new treatment of chiral perturbation theory has a negligible effect on the values of the matrix elements in this analysis.

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